Two-dimensional fin efficiency with combined heat and mass transfer between water-wetted fin surface and moving moist airstream

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A two-dimensional model has been developed to analyze the fin performance with combined heat and mass transfer in cooling and dehumidifying processes. The effect of the variations in moist air temperature and humidity on fin performance has been taken into account. The present results vary significantly, especially at low relative humidities, from those of the existing one-dimensional model. This may prove that the assumption of a lumped parameter to model the mass transfer effect is perhaps oversimplified. By means of this model, the fin temperature distribution, fin efficiency, and the properties of air in contact with the fin surface are predicted. The two-dimensional results show that the streamwise variations in moist air properties have a substantial effect on the fin performance and the trend is to further reduce fin efficiency. A nonuniform transverse distribution of outlet air properties is predicted. In other words, the air path line on the psychrometric chart along the fin base is not the same as that along the fin tip, while a simple straight air path line was assumed in the development of conventional one-dimensional fin model. It is also shown that, for a given inlet air dry bulb temperature, partial condensation occurred on the fin surface at low relative humidities, and a totally dry fin surface is formed at certain low relative humidities.

Keywords: condensation; plate fin; fin efficiency; cooling; dehumidification

Introduction

When the heat exchanger surface in contact with moist air is at a temperature below the dew point temperature for the air, condensation of water vapor will occur. The fin efficiency of a wet fin with condensation is lower than that of a dry fin due to the additional effect of condensation on the surface. Several attempts have been made to analyze fin efficiency with condensation from moist air. Threlkeld[1] assumed that a moving film of water was formed on the surface by condensation of moisture from the airstream. He made a conclusion that the solutions for efficiency of dry fins also apply for efficiency of wet fins if the heat transfer coefficient for the dry fin is replaced by a modified heat transfer coefficient in which the water film thermal resistance is taken into account. However, his solution depends on knowing the water film thickness on the fin. An accepted method is outlined in ARI Standard 410-72[2] and is an adaptation of the work of Ware and Hacha.[3] The coil surface temperature is assumed to be the only parameter affecting fin efficiency regardless of the moist air conditions. Another disturbing feature is failure of the solution to reduce to the dry coil case when the surface and moist air conditions warrant this. A fin of uniform cross section has been analyzed by McQuiston.[4] The method is approximate but reduces to the case of zero mass transfer. A single straight air path line between the inlet and exit conditions on the psychrometric chart was assumed in his analysis. The effect of mass transfer was taken into account by the slope of this air path line, which was taken to be a constant determined from the conditions of surrounding moist air and conditions at fin base. Although this solution can reduce to the case of zero mass transfer, the effect of local fin temperature on the slope of air path line was ignored. Recently, Coney et al.[5] applied a heat and mass transfer analogy to the conjugation of convected heat and mass in a buffer layer adjacent to the condensate layer, with heat conduction through the fin. The fin temperature distribution, the condensate film thickness, and the fin efficiency were predicted numerically for a fin in a laminar humid air cross flow. All of these were based on the assumption of a one-dimensional (1-D) fin with uniform environmental parameters (air temperature and humidity) over the fin surface. However, because of the simultaneous heat and mass transfer between the water-wetted fin surface and air in the airflow direction, the local environmental parameters do vary over the fin surface. To take into account the effect of varying environmental parameters on fin efficiency, a two-dimensional (2-D) analysis is necessary. The purpose of this article is to develop a 2-D theoretical model of a flat fin in contact with moving moist airstream. It is believed that the 2-D analysis will enable a more accurate evaluation to be made of fin efficiency in cooling and dehumidifying processes and provide insight into answers to many of the questions left unanswered by the 1-D analysis.

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Received 27 April 1990; accepted 26 July 1990

Analysis

A steady-state analysis is made on a fin system consisting of a simple flat fin and adjacent moving moist airstream, as shown in Figure 1. The following assumptions are made to simplify the analysis.

(1) The thermal conductivity of the fin, heat transfer coefficient, and humid specific heat of the airstream are constant.
(2) The moist air flow is steady with uniform velocities.
(3) The thermal resistance associated with the presence of thin water film due to condensation is small and may be neglected.
(4) The air is at 1 atm, and the effect of air pressure drop due to air flow is neglected.

Energy balances are made on the fin plate and the moving moist airstream separately to yield the governing differential equations.

If the fin plate temperature distribution is assumed to be 2-D then the governing differential equation for the fin plate becomes

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{2q}{kt}=0 \tag{1}$$

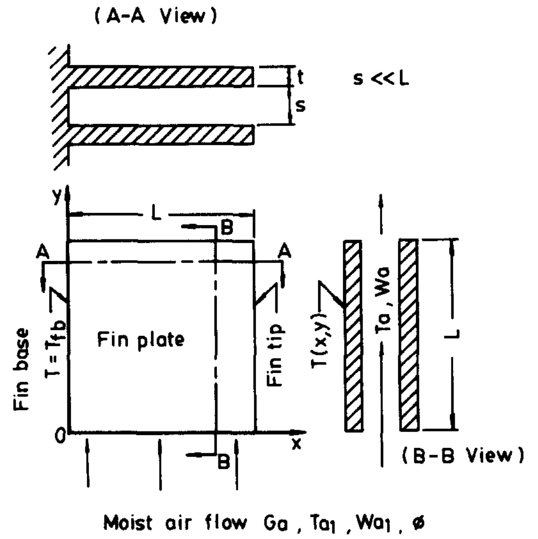

Figure 1 The schematic diagram of the fin system

where k is the thermal conductivity of the fin material, t is the plate thickness, q is the local heat flux at the fin surface, and T is the fin temperature.

The heat flux, q, from the moist air to the fin plate has two parts:

(1) sensible heat transfer, q_s, due to the temperature difference between the fin surface and the moist airstream;
(2) latent heat transfer, q_L, due to condensation.

Hence

$$q=q_s+q_L \tag{2}$$

The mass transfer coefficient is defined using the humidity ratio, W, as the driving potential:

$$\dot{m}=K_m(W_a-W) \tag{3}$$

where $\dot{m}$ is the mass flux of water vapor condensed on the surface and K_m is the mass transfer coefficient. Now, the water vapor condensed on the fin surface must be supplied with its latent heat of condensation, h_{fg}. Hence the resultant heat flux at the interface is

$$q_L=K_m\cdot h_{fg}\cdot(W_a-W) \tag{4}$$

With the assumption that the Lewis number for moist air is 1, Equation 4 may be written as

$$q_L=h_c\cdot h_{fg}\cdot\left(\frac{W_a-W}{C_{pm}}\right) \tag{5}$$

where h_c is the convective heat transfer coefficient and C_{pm} is the humid specific heat of the airstream. The convective sensible heat flux may be written as

$$q_s=h_c\cdot(T_a-T) \tag{6}$$

Thus from equations 2, 5, and 6, the total heat flux from the moist air to the fin is

$$q=h_c(T_a-T)+h_{fg}\left(\frac{W_a-W}{C_{pm}}\right) \tag{7}$$

where T_a is the air temperature, W_a is the humidity ratio of the moist airstream, T is the fin temperature, and W is the humidity ratio of the saturated air at temperature T.

An energy balance on the moving moist airstream in contact with a differential fin surface area, (dx/dy), yields

Notation

A	Coefficient, Equation 32
B	Coefficient, Equation 33
C	Variable, condensation factor
C_{pm}	Specific heat of moist air, J/Kg K
G_a	Mass flux of air, Kg/sec m^2
h_c	Heat transfer coefficient, W/m^2 K
h_{fg}	Latent heat of evaporation, J/Kg
k	Fin material thermal conductivity, W/m K
K_m	Mass transfer coefficient, Kg/sec m^2
L	Length and width of the fin, m
M	Fin parameter, Equation 23
$\dot{m}$	Mass flux of condensed vapor, Kg/sec m^2
N	Number of heat transfer units, Equation 24
p	Coefficient, Equation 31
q	Local heat flux at fin surface, W/m^2
q_L	Local latent heat flux, W/m^2
q_s	Local sensible heat flux, W/m^2
q_y	Local heat flux conducted to the fin base, W/m^2
$\dot{Q}_i$	Overall rate of heat transfer for an ideal fin, W
s	Fin spacing, m
t	Fin thickness, m
T	Fin temperature, K
T_a	Air temperature, K
W	Humidity ratio of saturated air at T
W_a	Humidity ratio of air
x	Transverse distance from fin base, m
y	Streamwise distance from inlet, m

Subscripts

0	At $x=0$ and $y=0$
1	At inlet ($y=0$)
fb	At fin base ($x=0$)

Superscripts

*	Dimensionless

(1) mass transfer

$$\frac{\partial W_a}{\partial y}=-\frac{2h_c}{G_a s C_{pm}}(W_a-W) \tag{8}$$

(2) heat transfer to the fin surface

$$\frac{\partial T_a}{\partial y}=-\frac{2h_c}{G_a s C_{pm}}(T_a-T) \tag{9}$$

where G_a is the steady mass flow rate of dry air per unit of cross-sectional area, or mass velocity for the air, and s is the fin spacing. Combining Equations 8 and 9 gives

$$\frac{\partial W_a}{\partial T_a}=\frac{W_a-W}{T_a-T} \tag{10}$$

Equation 10 indicates that at any position on the fin surface, the instantaneous slope of the air path, $\partial W_a/\partial T_a$, on a psychrometric chart is determined by a straight line connecting the air state with the interface saturation state at that position. If we denote the slope by a coefficient C, we may write

$$C=\frac{W_a-W}{T_a-T} \tag{11}$$

Now, from Equations 1, 7, and 11,

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{2h_c}{kt}(T_a-T)\left(1+\frac{h_{fg}C}{C_{pm}}\right)=0 \tag{12}$$

Equations 8, 9, and 12 are the governing differential equations resulting from this analysis. When solved, these equations give both the fin temperature distribution and the air property distribution over the fin surface. During the condensation of water vapor from moist air, the effect of the mass transfer on the total amount of heat liberated can be indicated by the term $h_{fg}C/C_{pm}$, as given by Equation 12. In the model of McQuiston a constant value of $C=(W_{a_1}-W_{fb})/(T_{a_1}-T_{fb})$ was assumed, while in the present analysis the local value of C is computed by using Equation 11.

The value of W is related to T by a second-degree polynomial relationship, which was derived in Reference 5:

$$W=1.2075-0.009125\cdot T+0.00001726\cdot T^2 \tag{13}$$

The boundary conditions utilized are insulated plate edges on all sides except along the fin base. Thus we have

$$T=T_{fb} \qquad \text{at } x=0 \tag{14}$$

$$\frac{\partial T}{\partial x}=0 \qquad \text{at } x=L \tag{15}$$

$$\frac{\partial T}{\partial y}=0 \qquad \text{at } y=0 \tag{16}$$

$$\frac{\partial T}{\partial y}=0 \qquad \text{at } y=L \tag{17}$$

The inlet boundary conditions for the moist airstream are

$$T_a=T_{a_1} \qquad \text{at } y=0 \tag{18}$$

$$W_a=W_{a_1} \qquad \text{at } y=0 \tag{19}$$

The governing differential equations are recast in dimensionless form by introducing the following expressions:

- dimensionless distance

$$x^*=\frac{x}{L} \qquad y^*=\frac{y}{L} \tag{20}$$

- dimensionless temperature

$$T^*=\frac{T-T_{fb}}{T_{a_1}-T_{fb}} \qquad T_a^*=\frac{T_a-T_{fb}}{T_{a_1}-T_{fb}} \tag{21}$$

- dimensionless humidity ratio

$$W^*=\frac{W-W_{fb}}{W_{a_1}-W_{fb}} \qquad W_a^*=\frac{W_a-W_{fb}}{W_{a_1}-W_{fb}} \tag{22}$$

- fin parameter

$$M=\frac{2h_cL^2}{kt} \tag{23}$$

- number of heat transfer units

$$N=\frac{2h_cL}{G_a s C_{pm}} \tag{24}$$

- dimensionless condensation factor

$$C^*=\frac{h_{fg}C}{C_{pm}} \tag{25}$$

When Equations 10–25 are used, we then obtain

$$\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}+M(T_a^*-T^*)(1+C^*)=0 \tag{26}$$

$$\frac{\partial T_a^*}{\partial y^*}=-N(T_a^*-T^*) \tag{27}$$

and

$$\frac{\partial W_a^*}{\partial y^*}=-N(W_a^*-W^*) \tag{28}$$

where

$$C^*=P\cdot\frac{W_a^*-W^*}{T_a^*-T^*} \tag{29}$$

$$W^*=AT^*+BT^{*2} \tag{30}$$

$$P=\frac{h_{fg}(W_{a_1}-W_{fb})}{C_{pm}(T_{a_1}-T_{fb})} \tag{31}$$

$$A=\frac{(T_{a_1}-T_{fb})}{(W_{a_1}-W_{fb})}(-0.009125+2\cdot 1.726\cdot 10^{-5}T_{fb}) \tag{32}$$

$$B=\frac{(T_{a_1}-T_{fb})^2}{(W_{a_1}-W_{fb})}(1.726\cdot 10^{-5}) \tag{33}$$

The associated boundary conditions become

$$\frac{\partial T^*}{\partial x^*}=0 \qquad \text{at } x^*=1 \tag{34}$$

$$\frac{\partial T^*}{\partial y^*}=0 \qquad W_a^*=1 \text{ and } T_a^*=a \qquad \text{at } y^*=0 \tag{35}$$

$$\frac{\partial T^*}{\partial y^*}=0 \qquad \text{at } y^*=1 \tag{36}$$

$$T^*=0 \qquad \text{at } x^*=0 \tag{37}$$

At any value of y, the local heat flux conducted from the fin to its base surface is given by Fourier's law,

$$q_y=k\left.\frac{\partial T}{\partial x}\right|_{x=0} \tag{38}$$

or, in terms of the dimensionless variable,

$$q_y = \frac{k(t_{a_1} - T_{fb})}{L} \frac{\partial T^*}{\partial x^*}\bigg|_{x^*=0} \tag{39}$$

The rate of heat transfer from the moist air to an ideal strip of fin with uniform temperature, T_{fb}, at y with a surface area $L\,dy$ is

$$\dot{Q}_i = 2h_c\left[1 + \frac{h_{fg}(W_{a_1} - W_{fb})}{C_{pm}(T_{a_1} - T_{fb})}\right](T_{a_1} - T_{fb})(L\,dy) \tag{40}$$

We may define local fin efficiency, η_y, as

$$\eta_y = \frac{q_y t\,dy}{\dot{Q}_i} = \frac{\dfrac{k(T_{a_1} - T_{fb})}{L}\dfrac{\partial T^*}{\partial x^*}\bigg|_{x^*=0} t\,dy}{2h_c(1 + C_0^*)(T_{a_1} - T_{fb})L\,dy} \tag{41}$$

or

$$\eta_y = \frac{\dfrac{\partial T^*}{\partial x^*}\bigg|_{x^*=0}}{M(1 + C_0^*)} \tag{42}$$

where

$$C_0^* = \frac{h_{fg}(W_{a_1} - W_{fb})}{C_{pm}(T_{a_1} - T_{fb})}$$

is the nondimensional condensation factor evaluated at inlet and fin base conditions.

The overall fin performance can be expressed by an overall fin efficiency, defined as the ratio of heat transfer from the fin to its base over a width L to the heat transfer from the moist air to an ideal fin:

$$\eta = \frac{kt(T_{a_1} - T_{fb})\displaystyle\int_0^1 \frac{\partial T^*}{\partial x^*}\bigg|_{x^*=0} dy^*}{2h_c(1 + C_0^*)(T_{a_1} - T_{fb})L^2} \tag{43}$$

or

$$\eta = \frac{\displaystyle\int_0^1 \frac{\partial T^*}{\partial x^*}\bigg|_{x^*=0} dy^*}{M(1 + C_0^*)} \tag{44}$$

The three differential equations, Equations 26, 27, and 28, together with the boundary conditions, Equations 34–37, are solved by an iterative numerical integration scheme.

Equation 26 is written in a differential-difference form in which the second-order derivative $\partial^2 T^*/\partial y^{*2}$ is replaced by central difference. The resulting differential-difference equation is integrated by means of the fourth-order Runge–Kutta technique. To start the integration at any y^*, an assumed value of $\partial T^*/\partial x^*$ at $x^* = 0$ is made. The integration is then carried out by using the Runge–Kutta algorithm between $x^* = 0$ and the tip of the fin at $x^* = 1$. The Newton–Raphson iteration technique is used to improve the value of $\partial T^*/\partial x^*$ at $x^* = 0$ until the adiabatic boundary condition at $x^* = 1$ is satisfied. This numerical procedure coupled with the calculation of moist air properties by using Equations 27 and 28 is applied from $y^* = 0$ to $y^* = 1$ and repeated until converged solutions are obtained.

Results and discussion

All of the theoretical studies were carried out for the model square flat fin considered by McQuiston in his analysis. The corresponding value of the fin parameter, M, is 0.468, with fin base temperature, T_{fb}, 280 K, humid air inlet temperature, T_{a_1}, 300 K, and air relative humidity, ϕ, range of 20%–70%. Three values of 0, 0.5, and 1.0 for the number of heat transfer units, N, are chosen. The results presented in this article are all based on these data.

We may see from Equations 27 and 28 that if $N = 0$ the moist air properties remain unchanged in the flow direction. In other words, the fin is surrounded by air with a uniform temperature and humidity ratio and is reduced to the 1-D case. Thus the present results with $N = 0$ are the 1-D fin solutions of the present model. The 1-D fin temperature distribution together with that computed by using the model of McQuiston for $\phi = 0.3$ and $\phi = 0.7$ is shown in Figure 2. The 1-D fin efficiency with various values of air relative humidity, ϕ, is presented and compared with other models in Figure 3. The nondimensional condensation factor, C^*, distribution over the fin length is plotted in Figure 4. The results of the present model, as illustrated by these figures, vary significantly from those of the model developed by McQuiston. The deviations in the results between these two models appear due to the different treatment of the factor C in

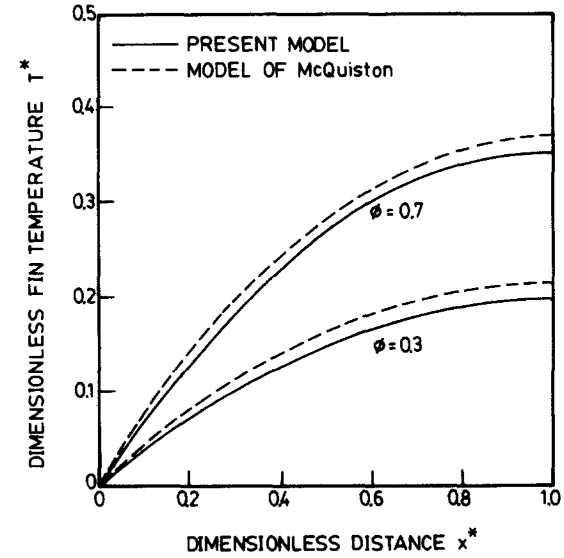

Figure 2 One-dimensional fin temperature for $\phi = 0.3$ and $\phi = 0.7$

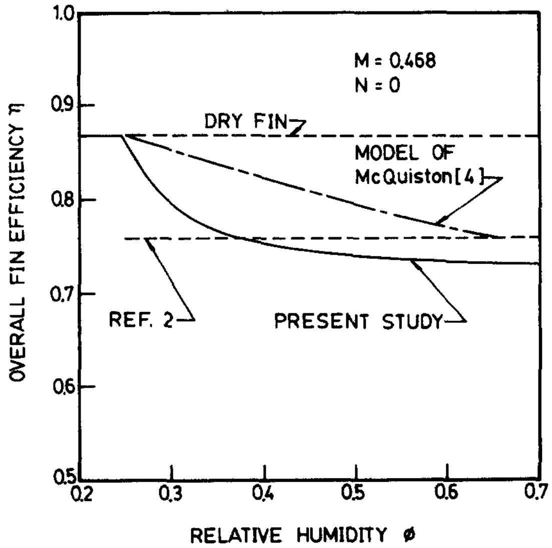

Figure 3 One-dimensional fin efficiency

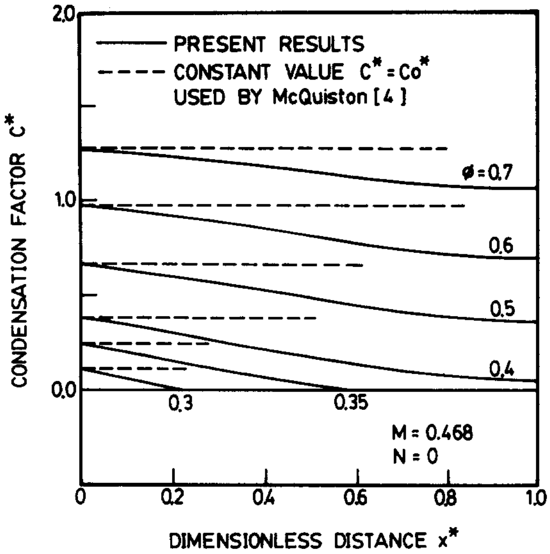

Figure 4 One-dimensional fin condensation factor

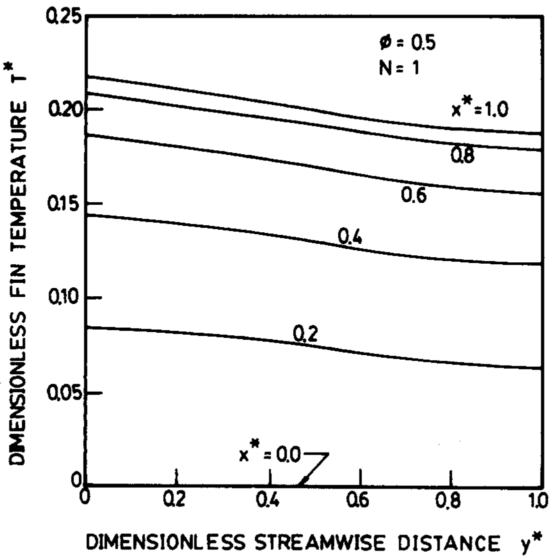

Figure 5 Two-dimensional fin temperature for $\phi = 0.5$ and $N = 1$

the differential equation describing the fin temperature distribution. A constant value of C was assumed in the model of McQuiston while a variable C is allowed to be computed in the present model. For simultaneous heat and mass transfer between wet surface and moist air, the value of C equals the instantaneous slope of the air path line on a psychrometric chart (by Equation 10). McQuiston assumed the air path line between the inlet and exit conditions on the psychrometric chart to be straight and the corresponding C to be independent of x. However, because of the fin temperature variations in the x direction, the local value of C does vary over the surface. The simplifying assumption made by McQuiston seems to be valid only when the driving potential difference for mass transfer, $W_a - W$, is large such that the small variation in W associated with varying T can be neglected. The effect of variable C becomes significant for reduced $W_a - W$ at low ϕ as shown by these figures. It shows that the value of C decreases with increasing x and decreasing ϕ. For the case shown in Figure 4, the fin tip becomes dry (with $C = 0$) at $\phi \simeq 0.37$. A partially dry fin is observed analytically for further decrease in ϕ, and a totally dry fin appears when $\phi \simeq 0.27$. The fin efficiency, as shown in Figure 3 and calculated by the present model, compares well with the results of Reference 2. This may prove important in considering the local effect of the presence of condensation on fin performance.

Further consideration of the local condensation effect in the y direction is achieved by performing a 2-D analysis. The 2-D fin temperature distribution is presented in Figure 5. The results are obtained for relative humidity $\phi = 0.5$ and a number of heat transfer units $N = 1.0$. As expected, the fin temperatures change from the inlet to the exit because of the presence of streamwise variations in moist air properties. The distribution of moist air properties over the fin surface is shown in Figure 6. Again, as one would expect, the model shows decreasing relative humidity and temperature with increasing streamwise distance. It is also shown that, at a given y^*, the values of W_a^* and T_a^* increase with increasing distance x^*. The local condensation rate is illustrated best by the distribution of C^* over the fin surface. The results are shown in Figure 7. Due to the maximum driving potential difference at the fin base, the value of C^* is maximum

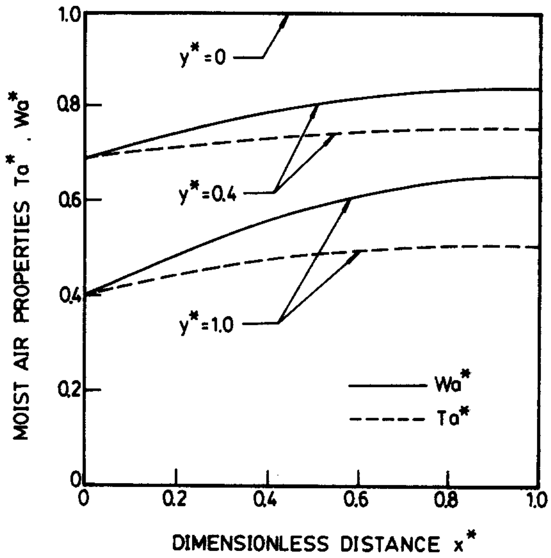

Figure 6 Two-dimensional fin air properties for $\phi = 0.5$ and $N = 1$

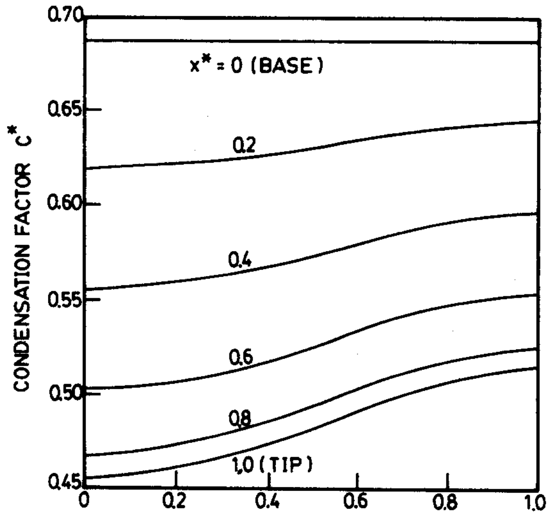

Figure 7 Two-dimensional fin condensation factor for $\phi = 0.5$ and $N = 1$

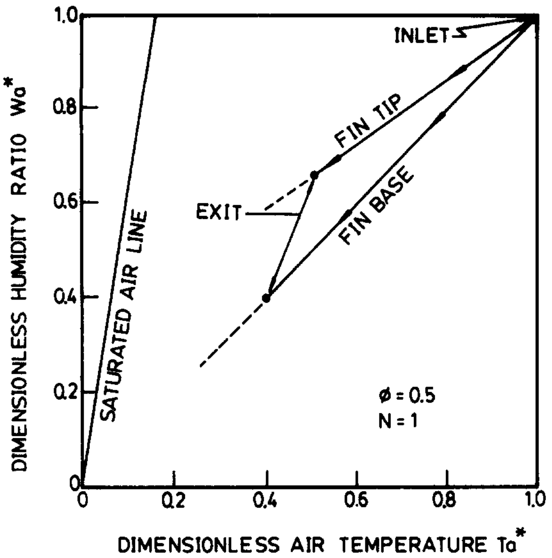

Figure 8 Two-dimensional fin air path lines along $x^*=0$ and $x^*=1$ for $\phi=0.5$ and $N=1$

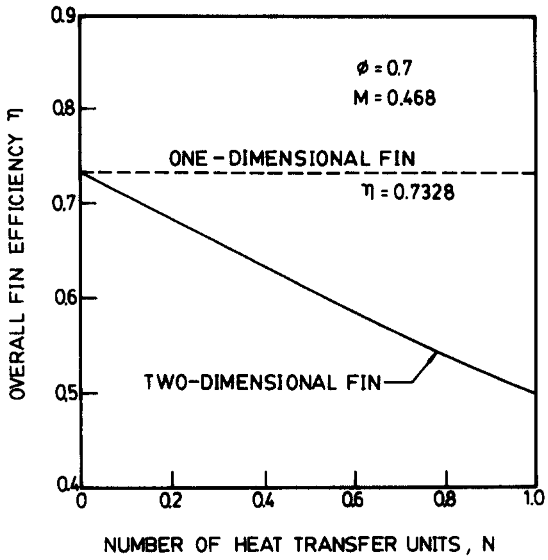

Figure 10 Two-dimensional fin overall fin efficiency for $\phi=0.7$

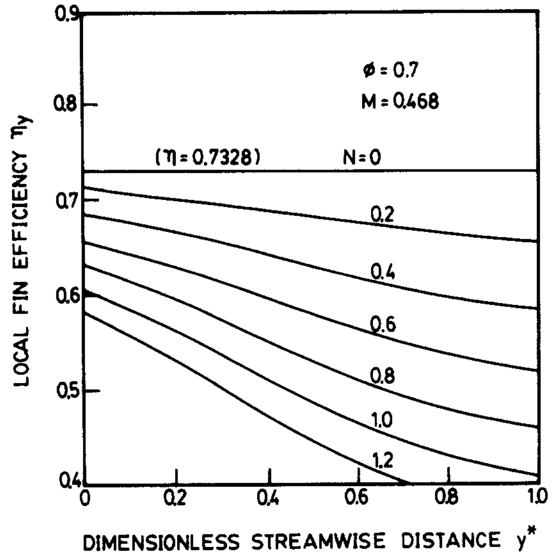

Figure 9 Two-dimensional fin local fin efficiency for $\phi=0.7$

at $x^*=0$. Figure 7 also illustrates an important feature of the solutions obtained for C^*, which are different from the constant value. This variation is indicated in both the x and y directions. Figure 8 is of some interest as it shows the air path lines along the fin base and the fin tip. The air path lines are constructed by plotting W_a^* against T_a^* at each point (x^*, y^*) along the fin base and the fin tip on a "reduced" psychrometric chart in which the saturated air line is expressed in terms of W_s^* and T_a^* at saturation. The results are given by the lines in Figure 8, from which it appears that the air path over the fin surface is represented by a straight line with a certain slope on the chart. The slope of the air path line along the fin tip is different from that of the air path line along the fin base due to different heat and mass transfer rates. The effect of a varying environmental parameter on the fin performance is illustrated in Figure 9. It is interesting to note that the 2-D fin result reduces to the 1-D fin case when N reduces to zero, as expected. The local fin efficiency decreases with increasing N and y^*. The overall 2-D effects of varying environmental parameters on the fin performance are demonstrated in Figure 10. Again, as one would expect, the model shows decreasing overall fin efficiency with increasing N.

Conclusions

In this article, some of the important effects of the fin and air parameters on the fin performance have been presented. The approach used in this study does not require assumption of a lumped condensation factor. The results of this model vary significantly from those of the traditional lumped 1-D model, which, as one should expect, give higher efficiencies in certain operating conditions. This may prove important in evaluating the fin performance with varying environmental parameters over the fin surface in the cooling and dehumidifying process.

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